ATME College of Engineering

Control Systems BEC403
IV Semester

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VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELAGAVI
B.E: Electronics & Communication Engineering / B.E: Electronics & Telecommunication EngineeringNEP, Outcome Based Education (OBE) and Choice Based Credit System (CBCS)
(Effective from the academic year 2023 – 24)

IV Semester

Control Systems			
Course Code	**BEC403**	CIE Marks	50
Teaching Hours/Week (L: T: P)	(3:0:2)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 12 Lab slots	Total Marks	100
Credits	04	Exam Hours	03

Course objectives: This course will enable students to:

1. Understand basics of control systems and design mathematical models using block diagram reduction, SFG, etc.
2. Understand Time domain and Frequency domain analysis.
3. Analyze the stability of a system from the transfer function
4. Familiarize with the State Space Model of the system.

Module-1	
Introduction to Control Systems: Types of Control Systems, Effect of Feedback Systems, Differential equation of Physical Systems -Mechanical Systems, Electrical Systems, Analogous Systems. (Textbook 1: Chapter 1.1, 2.2)	
Teaching-LearningProcess	Chalk and Talk, YouTube videos **RBT Level:** L1, L2, L3

Module-2	
Block diagrams and signal flow graphs: Transfer functions, Block diagram algebra and Signal Flow graphs. (Textbook 1: Chapter 2.4, 2.5, 2.6)	
Teaching-LearningProcess	Chalk and Talk, YouTube videos, Any software tool to implement block diagram reduction techniques and Signal Flow graphs **RBT Level:** L1, L2, L3
Module-3	
Time Response of feedback control systems: Standard test signals, Unit step response of First and Second order Systems. Time response specifications, Time response specifications of second order systems, steady state errors and error constants. Introduction to PI, PD and PID Controllers (excluding design). (Textbook 1: Chapter 5.3, 5.4, 5.5)	
Teaching-LearningProcess	Chalk and Talk, YouTube videos, Any software tool to show time response for various transfer functions and PI, PD and PID controllers. **RBT Level:** L1, L2, L3

Module-4	
Stability analysis: Concepts of stability, Necessary conditions for Stability, Routh stability criterion, Relative stability analysis: more on the Routh stability criterion. **Introduction to Root-Locus Techniques**, The root locus concepts, Construction of root loci. (Textbook 1: Chapter 6.1, 6.2, 6.4, 6.5, 7.1, 7.2, 7.3)	
Teaching-LearningProcess	Chalk and Talk, YouTube videos, Any software tool to plot Root locus for various transfer functions **RBT Level:** L1, L2, L3
Module-5	
Frequency domain analysis and stability: Correlation between time and frequency response, Bode Plots, Experimental determination of transfer function. (Textbook 1: Chapter 4: 8.1, 8.2, 8.4) Mathematical preliminaries, Nyquist Stability criterion, (Stability criteria related to polar plots are excluded) (Textbook 1: 9.2, 9.3) **State Variable Analysis:** Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 1: 12.2, 12.3, 12.6)	

PRACTICAL COMPONENT OF IPCC

Using suitable simulation software (P-Spice/ MATLAB / Python / Scilab / OCTAVE / LabVIEW) demonstrate the operation of the following circuits:

Sl.No	Experiments
1	Implement Block diagram reduction technique to obtain transfer function a control system.
2	Implement Signal Flow graph to obtain transfer function a control system.
3	Simulation of poles and zeros of a transfer function.
4	Implement time response specification of a second order Under damped System, for different damping factors.
5	Implement frequency response of a second order System.
6	Implement frequency response of a lead lag compensator.
7	Analyze the stability of the given system using Routh stability criterion.
8	Analyze the stability of the given system using Root locus.
9	Analyze the stability of the given system using Bode plots.
10	Analyze the stability of the given system using Nyquist plot.
11	Obtain the time response from state model of a system.
12	Implement PI and PD Controllers.
13	Implement a PID Controller and hence realize an Error Detector.
14	Demonstrate the effect of PI, PD and PID controller on the system response.

Assessment Details (both CIE and SEE)

The IPCC means the practical portion integrated with the theory of the course. CIE marks for the theory component are **25 marks** and that for the practical component is **25 marks**.

CIE for the theory component of the IPCC

IPCC.

CIE for the practical component of the IPCC

- **15 marks** for the conduction of the experiment and preparation of laboratory record, and **10 marks** for the test to be conducted after the completion of all the laboratory sessions.

Suggested Learning Resources: **Text Books** 1. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.
Web links and Video Lectures (e-Resources): https://nptel.ac.in/courses/108106098
Activity Based Learning (Suggested Activities in Class)/ Practical Based learning *Programming Assignments / Mini Projects can be given to improve programming skills*

Why Control?

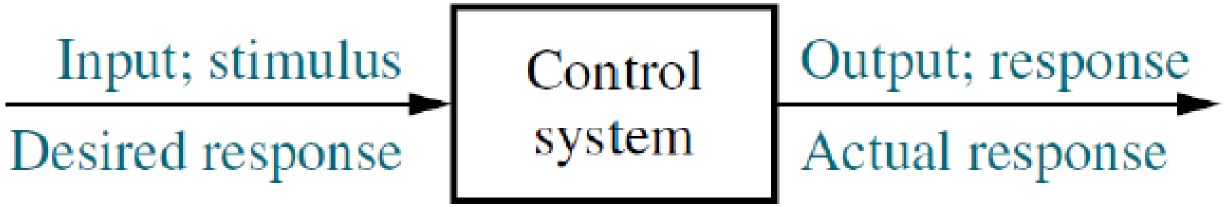
- Modern society have sophisticated control systems which are crucial to their successful operation.
- Reasons to build control systems:
  - Power amplification
  - Remote control
  - Convenience of input form
  - Compensation for disturbance

Control System Definition

System: Collection of elements arranged in a particular sequence that act together and perform an objective in a coordinated manner

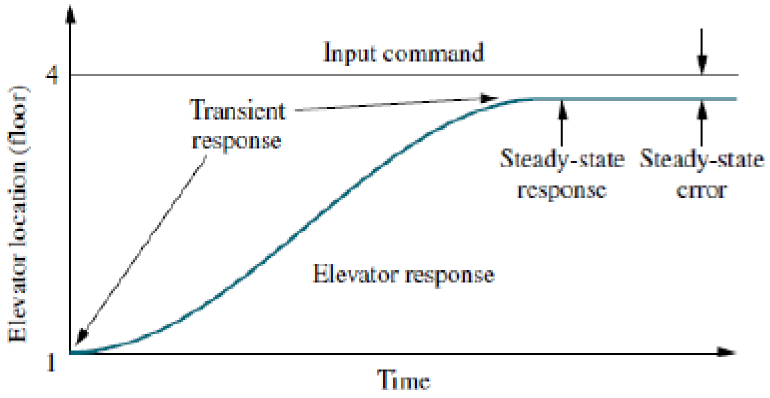
Control: To regulate, to command

Control System: A control system consists of subsystems and processes (or plants) assembled for the purpose of obtaining a desired output with desired performance, given a specified input.



Examples of Control System

- Simple ON – OFF control
- Central Temperature Control
- Lift Control System
- Battery Voltage Control (BMS in EV)
- Process Control
- Human Like Control (Robotics)

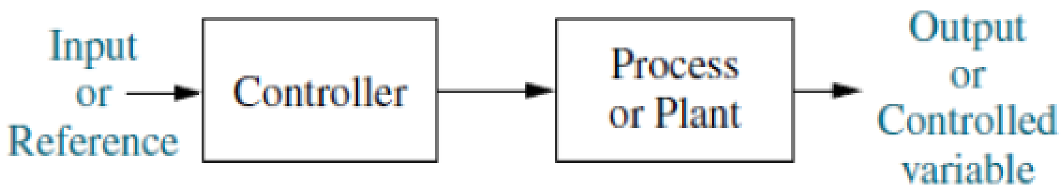


Advantages!!

1. Power amplification
2. Remote control
3. Convenience of input form
4. Compensation for disturbances

Control System - Classification

Open Loop System

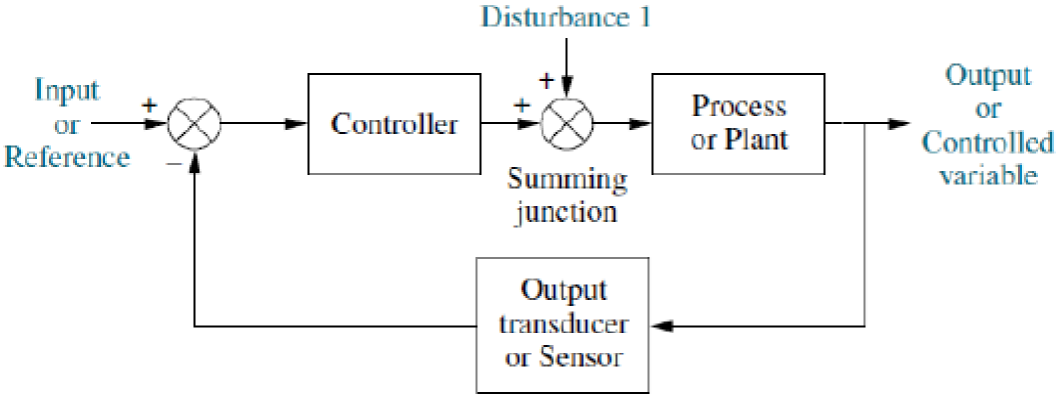


- Simple in design
- Economic & Easy Maintenance
- Convenient to use when output is difficult to measure
- Not much trouble with problems in stability
- ***Not accurate***
- ***Not Reliable***
- ***Recalibration is needed often***

Eg. Immersion heater, Toaster

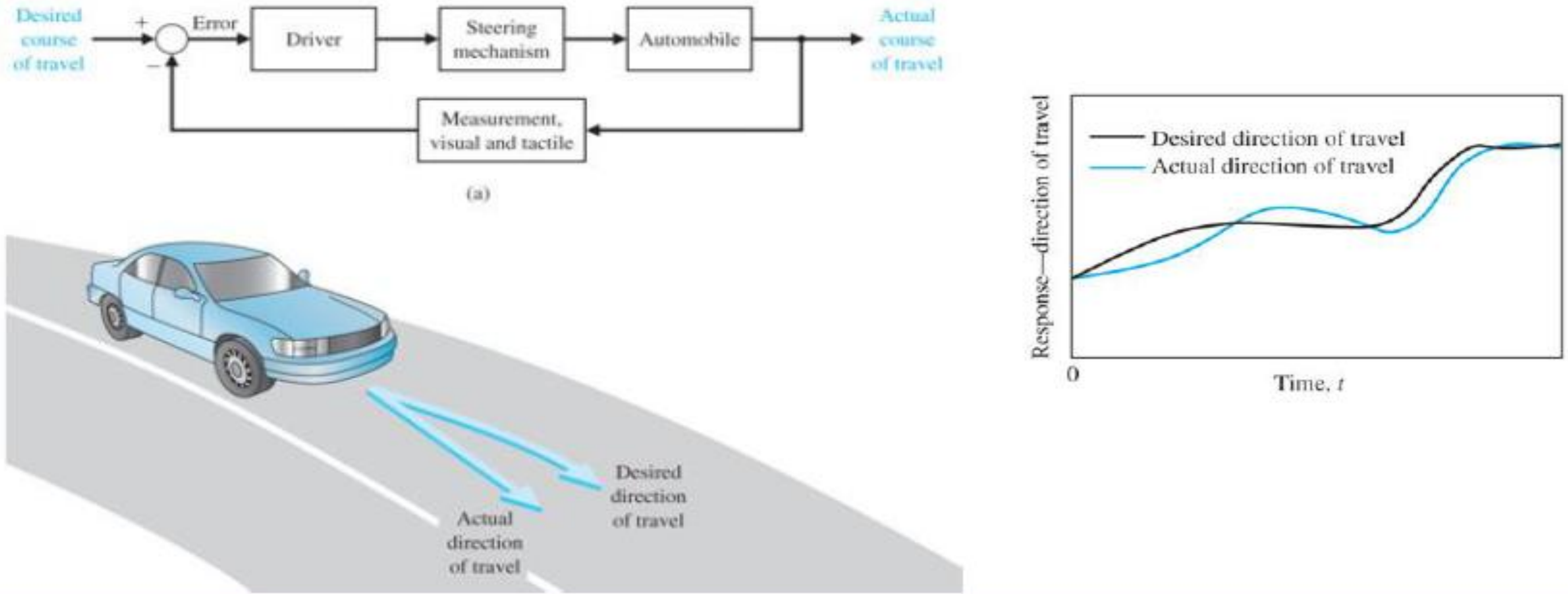
Control System - Classification

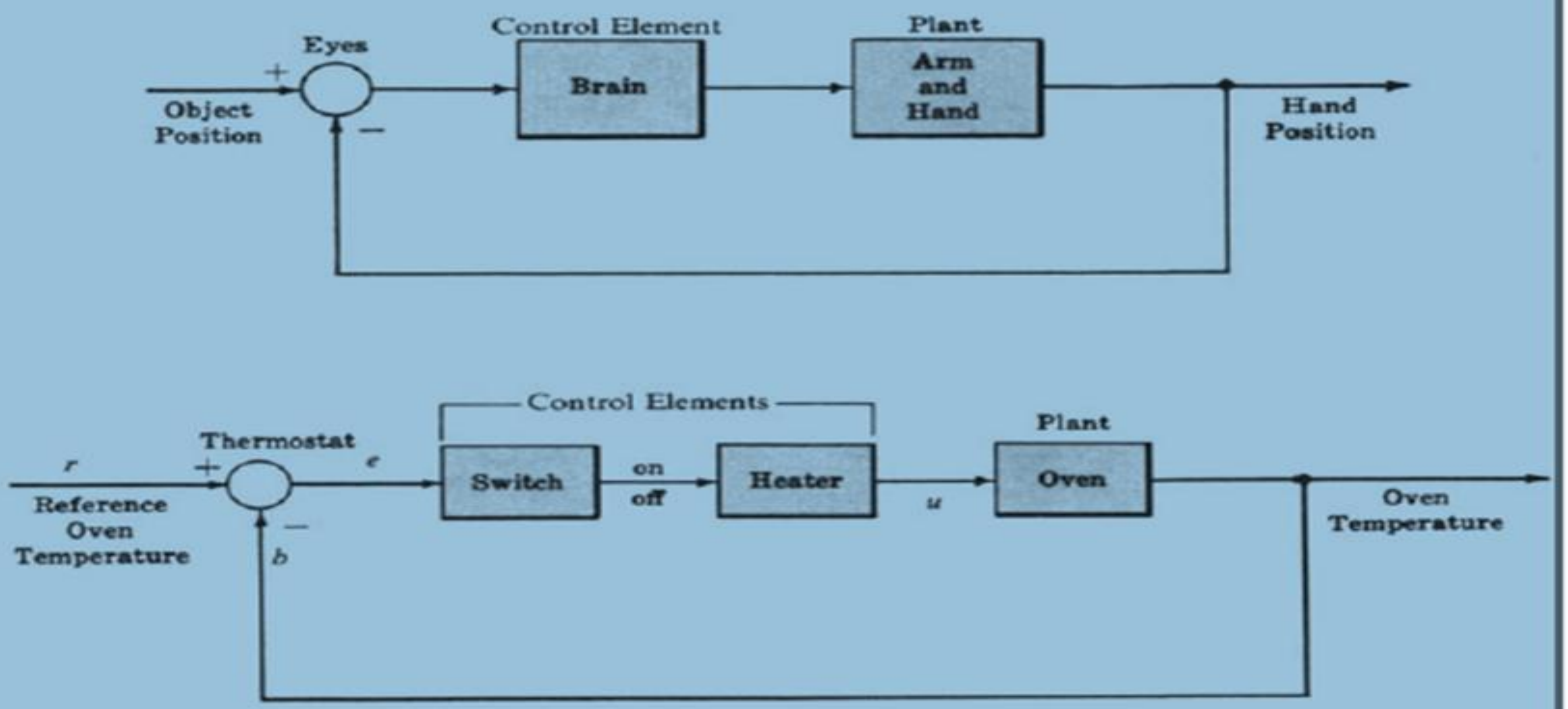
Closed Loop System



Eg. Centralized temperature control

Closed loop System/ Feedback System





When $e > 0$ ($r > b$), the switch turns the heater on. When $e \leq 0$, the heater is turned off.

modelling

- Any physical system can modelled into equivalent mathematical model
- Using basic physical laws such as ohms law and kirchoffs law in ele ctrical system
- Use Newtons second law assuming zero gravitational in mechanical system

The **transfer function of a linear, time-invariant,** differential equation system is given by the ratio of Laplace transform of output variable (response function) to the Laplace transform of the input variable (driving function) under the assumption that all initial conditions are zero.

$$\text{Transfer function, } G(s) = \frac{\mathcal{L}[c(t)]}{\mathcal{L}[r(t)]} = \frac{C(s)}{R(s)} \text{ at zero initial conditions.}$$

It is to be noted **that non-linear systems and time-varying systems do not have transfer functions** because they do not obey the principles of superposition and homogeneity

Features and Advantages of Transfer Function Representation

The following are **the features and advantages of transfer function representation:**

(i) Using transfer functions, mathematical models can be obtained and analyzed.

(ii) Output response can be obtained for any kind of inputs.

(iii) Stability analysis can be performed.

(iv) The usage of Laplace transform converts complex time domain equations to simple
algebraic equations, expressed with complex variable *s.*

(v) Analysis of a system is simplified due to the use of *s-domain variable in the equations,*
rather than using time-domain variable.

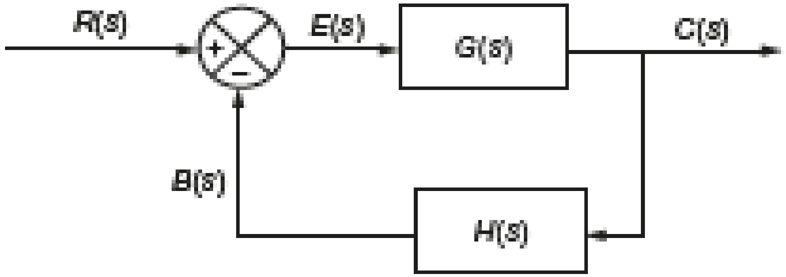
The following are the disadvantages of transfer function representation:
(i) It is not applicable to non-linear systems or time-varying systems.
(ii) Initial conditions are neglected.
(iii) Physical nature of a system cannot be found (i.e., whether it is mechanical or electrical or thermal system).

Transfer Function of a Closed-Loop System

Closed-loop control systems can be classified into two categories, based on the type of feedback signal as (i) Negative feedback systems and (ii) Positive feedback systems.

(i) Negative Feedback Systems

The simplest form of a negative-feedback control system is shown in Fig. 1.9. It has one block in the forward path and one in the feedback path.



Block diagram representation of negative feedback control system

The following relationships can be derived from the block diagram:

Feedback signal, $B(s)$ = output × feedback path gain

$$B(s) = C(s)H(s) \qquad (1.1)$$

Error signal, $E(s)$ = input signal – feedback signal

$$E(s) = R(s) - B(s)$$

Substituting Eqn. (1.1) in the above equation, we obtain

$$E(s) = R(s) - C(s)H(s) \qquad (1.2)$$

Output signal, $C(s)$ = error signal × forward path gain

$$C(s) = E(s)G(s) \qquad (1.3)$$

Substituting the value of the error signal, we obtain

$$C(s) = R(s)G(s) - C(s)G(s)H(s)$$

$$C(s) + C(s)G(s)H(s) = R(s)G(s)$$

$$C(s)\left(1 + G(s)H(s)\right) = R(s)G(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Hence, the transfer function of the negative feedback control system is given by

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \qquad (1.4)$$

Based on the above expression, Fig. 1.9 is reduced to the simplified form as shown in Fig. 1.10.

Fig. 1.10 | Reduced form of a negative feedback control system

Thus, the closed-loop transfer function is

$$T(s) = \frac{C(s)}{R(s)} = \frac{\text{forward path gain}}{1 + \text{forward path gain} \times \text{feedback path gain}}$$

Comparison of Positive Feedback and Negative Feedback Systems

Table 1.2 discusses the comparison of characteristics between positive feedback and negative feedback systems.

Table 1.2 | Characteristics of positive feedback and negative feedback systems

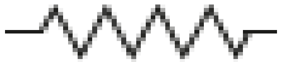
Characteristics	Negative feedback	Positive feedback
Stability	High	Low
Magnitude of transfer function	<1	>1
Sensitivity to parameter changes	Low	High
Gain	Low	High
Error signal	Decreased	Increased
Application	Motor control	Oscillators

Example 1.1: A negative feedback system has a forward gain of 10 and feedback gain of 1. Determine the overall gain of the system.

Modeling of Electrical Systems

An electrical system consists of resistors, capacitors and inductors. The differential equations of electrical systems can be formed by applying Kirchhoff's laws. The transfer function can be obtained by taking Laplace transform of the integro-differential equations and rearranging them as a ratio of output to input. The relationship between voltage and current for different elements in the electrical circuit is given in Table 1.3.

Table 1.3 | Relationship of voltage and current for R, L and C

Element	Voltage drop across the element	Current through the element
R 	$v(t) = Ri(t)$	$i(t) = \dfrac{v(t)}{R}$
L 	$v(t) = L\dfrac{di(t)}{dt}$	$i(t) = \dfrac{1}{L}\int v(t)dt$
C 	$v(t) = \dfrac{1}{C}\int i(t)dt$	$i(t) = C\dfrac{dv(t)}{dt}$

Example 1.5: Determine the transfer function of the electrical network shown in Fig. E1.5.

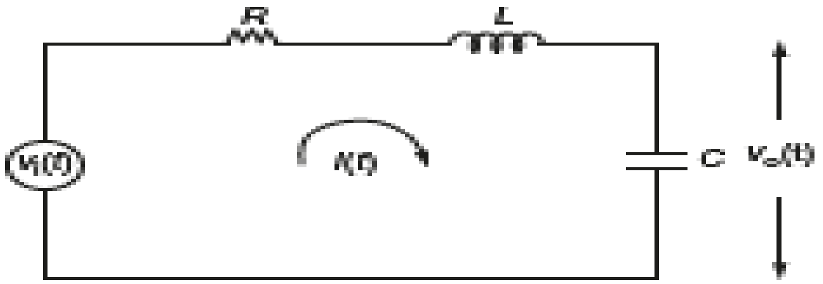


Fig. E1.5

Solution: The output voltage across the capacitor is $v_o(t) = \frac{1}{C}\int i(t)\,dt$

Applying Kirchhoff's voltage law, the loop equation is given by

$$v_i(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

Taking Laplace transform of the above equations, we obtain

$$V_o(s) = \frac{I(s)}{Cs} \qquad (1)$$

$$V_i(s) = \left(R + Ls + \frac{1}{Cs}\right)I(s)$$

Simplifying the above equation, we obtain

$$I(s) = \frac{V_i(s)}{\left(R + Ls + \frac{1}{Cs}\right)} \qquad (2)$$

Substituting Eqn. (2) in Eqn. (1), we obtain

$$V_o(s) = \frac{V_i(s)}{\left(R + Ls + \frac{1}{Cs}\right)} \times \frac{1}{Cs}$$

Hence, the transfer function of the system is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Modeling of Mechanical Systems

The dimensions in which the movement of a mechanical system can be described are translational, rotational or a combination of both. For modeling of mechanical system, it is necessary to have the equations governing the movement of mechanical systems. The laws that are used directly or indirectly to formulate those equations are obtained from Newton's laws of motion.

The general classification of mechanical system is of two types: (i) translational and (ii) rotational as shown in Fig. 1.13.

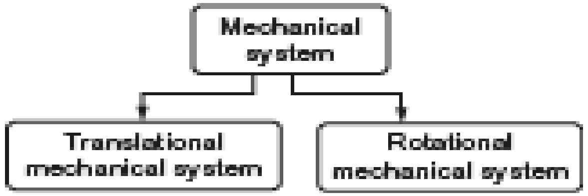


Fig. 1.13 | Classification of mechanical system

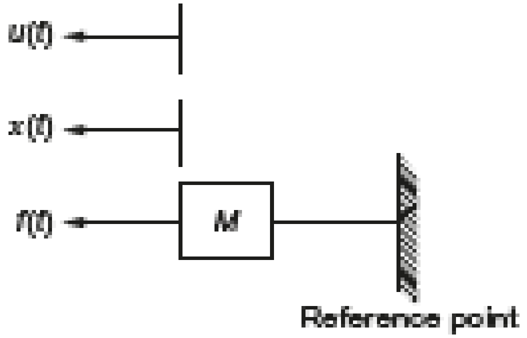


Fig. 1.14 | Mass element

According to Newton's second law, the force experienced by the mass is proportional to the acceleration.

$$f_M(t) \alpha \; a$$

$$f_M(t) \alpha \frac{du(t)}{dt}$$

$$f_M(t) = \text{Mass} \times \text{acceleration} = Ma$$

$$f_M(t) = M\frac{d^2x(t)}{dt^2} = M\frac{du(t)}{dt}$$

where M is the mass (kg), a is the acceleration (m/s²), $u(t)$ is the velocity (m/s) and $x(t)$ is the displacement (m).

$$f_M(t) = M\frac{d^2x(t)}{dt^2} = M\frac{du(t)}{dt} = Ma$$

Damper

The damping that is desirable in the motion for a mechanical system is provided by an element called damper. A frictional force that exists between one or two physical systems when there exists a movement or a tendency of movement between them. The characteristics of frictional forces that are non-linear in nature depends on the composition of the surfaces, the pressure between the surfaces, their relative velocity and others that add difficulty in obtaining the mathematical description of the frictional force. The different types of frictions that are commonly used in practical system are viscous friction, static friction and coulomb friction in which the viscous friction is commonly found in a translational mechanical system.

Static Friction or Stiction

The retarding force $f_S(t)$ which tends to prevent the motion is known as static friction or stiction. This type of frictional force exists only when the body is not in motion (stationary), but has the tendency to move. The sign or direction of static friction is opposite to the direction in which the body tends to move or initial direction of the velocity. This frictional force vanishes once the movement of a system is started.

Viscous Friction

The retarding force that is experienced by a system when it is in motion is known as viscous friction. This type of friction exists between a system and a fluid medium. There exists a linear relationship between the applied force and the velocity. Hence, this frictional force $f_B(t)$ is proportional to the velocity of the movement of a system.

$$f_B(t) \ \alpha \ \text{velocity}$$

This frictional force that is more common in a mechanical system is represented by a dashpot or a damper system. When a force is applied to a damping element B, it experiences a velocity and it is shown in Fig. 1.15(a).

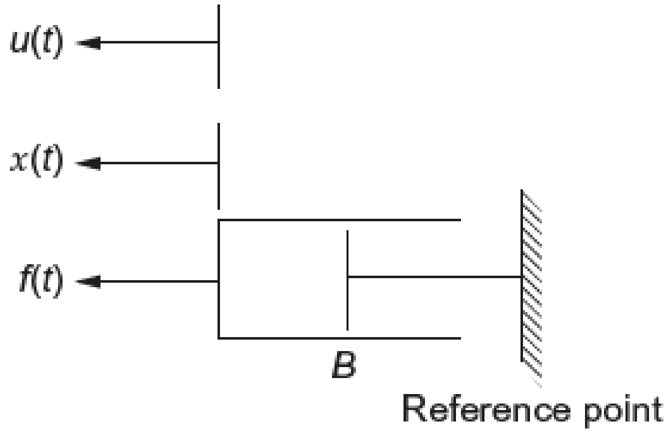


Fig. 1.15 | (a) Dash-pot or Damper element

$$f_B(t)\ \alpha\ u(t)$$

$$f_B(t) = Bu(t) = B\frac{dx(t)}{dt}$$

where B is the viscous friction coefficient (N-s/m), $u(t)$ is the velocity (m/s) and $x(t)$ is the displacement (m).

Dashpot or damper element with two displacements is shown in Fig. 1.15(b).

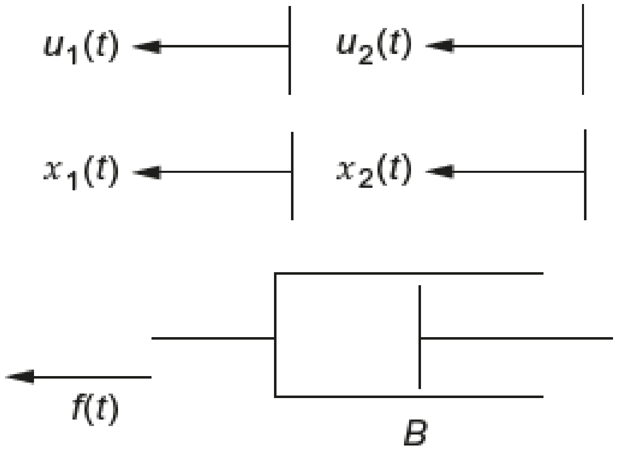


Fig. 1.15 | (b) Dash-pot or Damper element with different displacements

$$f_B(t) = B\left(u_1(t) - u_2(t)\right)$$

$$= B\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right)$$

Here, $f_B(t)$ is measured in Newton (N) or Kg-m/s².

Spring Force $f_K(t)$

In general, an element that stores potential energy in a system is known as spring. The analogous component of spring in an electrical circuit is the capacitor that is used to store voltages. In real time, the springs are non-linear in nature. However, if the spring deformation is small, the behaviour of the spring can be approximated by a linear relationship. The opposing force developed by the spring is directly related to the stiffness of the spring and the total displacement of the spring from its equilibrium position.

When a force $f(t)$ is applied to a spring element K, it experiences a displacement and it is shown in Fig. 1.17.

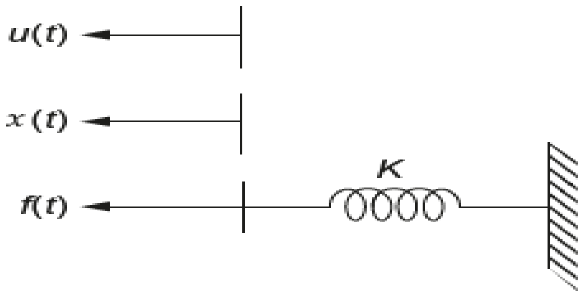


Fig. 1.17 | Spring element

According to Hooke's law, the spring force is directly proportional to the displacement:

$$f_K(t) \propto x(t)$$

The spring stores the potential energy. Therefore,

$$f_K(t) = K\,x(t)$$

Velocity,
$$u(t) = \frac{dx(t)}{dt}$$

$$dx(t) = u(t)\,dt$$

Integrating the above equation, we obtain

$$x(t) = \int u(t)\,dt$$

Therefore, $$f_K(t) = K\int u(t)\,dt$$

where K is the spring constant (N/m), i.e., $K = \dfrac{1}{\text{compliance}} = \dfrac{1}{k}$, $u(t)$ is the velocity (m/s) and $x(t)$ is the displacement (m).
Spring element with two displacements is shown in Fig. 1.18.

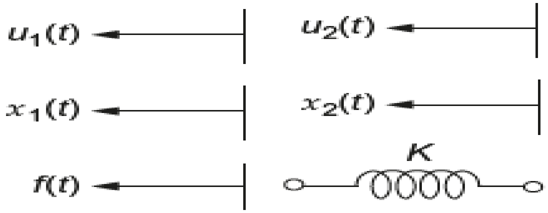


Fig. 1.18 | Spring element with two displacements

$$f_K(t) = K\left(x_1(t) - x_2(t)\right)$$

$$f_K(t) = \frac{1}{k}\left(x_1(t) - x_2(t)\right)$$

$$f_K(t) = \frac{1}{k}\int \left(u_1(t) - u_2(t)\right)dt$$

Here, $f_K(t)$ is measured in Newton (N) or Kg-m/s^2.

1.7.2 A Simple Translational Mechanical System

According to D'Alembert's principle, "The algebraic sum of the externally applied forces to any body is equal to the algebraic sum of the opposing forces restraining motion produced by the elements present in the body."

A simple translational mechanical system with all the basic elements and its free body diagram are shown in Figs. 1.19 (a) and (b) respectively.

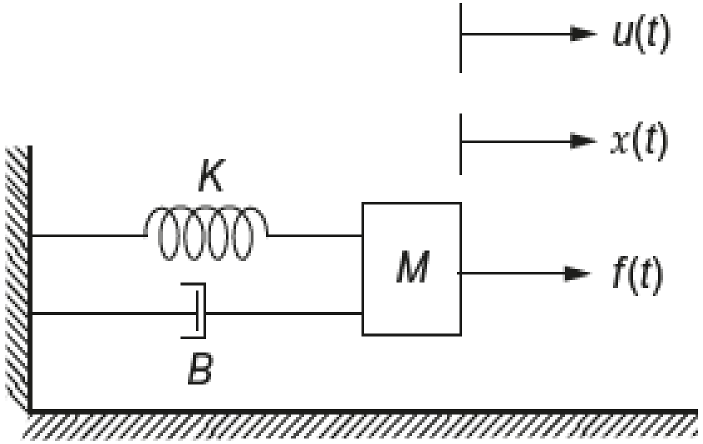


Fig. 1.19 | (a) A simple translational mechanical system

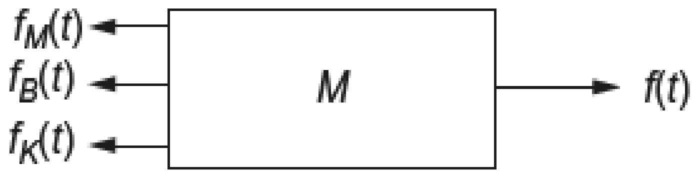


Fig. 1.19 | (b) Free body diagram

Let $f(t)$ be the external force applied to the given system,

$f_M(t) = M\dfrac{du(t)}{dt}$ be the opposing force produced by the element mass M,

$f_B(t) = B\,u(t)$ be the opposing force produced by the element damper B and

$f_K(t) = K\int u(t)\,dt$ be the opposing force produced by the element spring K.

Using D'Alembert's principle,

$$f(t) = f_M(t) + f_B(t) + f_K(t)$$

$$f(t) = M\frac{du(t)}{dt} + B\,u(t) + K\int u(t)\,dt$$

The above equation is referred to as *D'Alembert's basic equation for translational mechanical systems.*

Example 1.9: For the mechanical translational system shown in Fig. E1.9, obtain (i) differential equations and (ii) transfer function of the system.

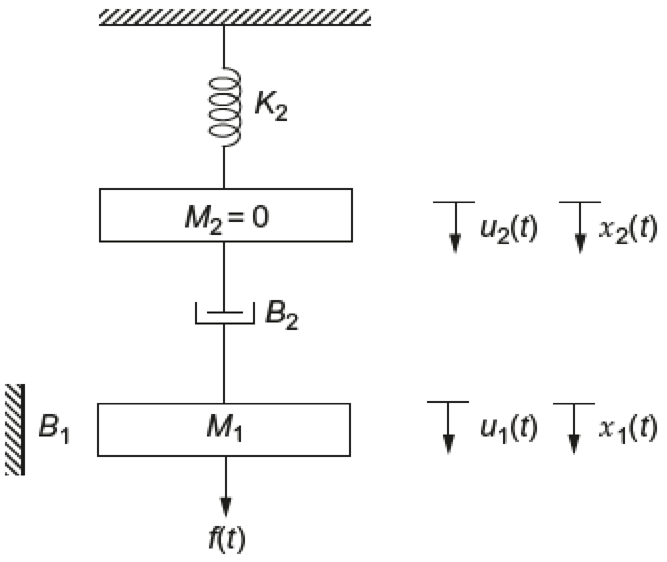


Fig. E1.9

Solution: For the system given in Fig. E1.9, the input is $f(t)$ and the outputs are $x_1(t)$ and $x_2(t)$. Using D' Alembert's principle for the mass M_1, we obtain

$$f(t) = f_{M1}(t) + f_{B1}(t) + f_{B2}(t)$$

$$f(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + B_2 \frac{d\left(x_1(t) - x_2(t)\right)}{dt} \qquad (1)$$

For the mass M_2, we obtain

$$0 = f_{M2}(t) + f_{B2}(t) + f_{K2}(t)$$

$$0 = B_2 \frac{d\left(x_2(t) - x_1(t)\right)}{dt} + K_2 x_2(t) \qquad (2)$$

Taking Laplace transform of Eqn. (1) and Eqn. (2), we obtain

$$\left[M_1 s^2 + (B_1 + B_2)s\right] X_1(s) - B_2 s X_2(s) = F(s)$$

$$-B_2 s X_1(s) + \{B_2 s + K_2\} X_2(s) = 0$$

Representing the above equations in matrix form,

$$\begin{bmatrix} M_1 s^2 + (B_1 + B_2)s & -B_2 s \\ -B_2 s & B_2 s + K_2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$$

Using Cramer's rule, we obtain

$$X_2(s)=\frac{\Delta_2}{\Delta} \text{ and } X_1(s)=\frac{\Delta_1}{\Delta}$$

$$\Delta_1=\begin{vmatrix} F(s) & -B_2s \\ 0 & B_2s+K_2 \end{vmatrix}=F(s)\times(B_2s+K_2)$$

$$\Delta_2=\begin{vmatrix} M_1s^2+(B_1+B_2)s & F(s) \\ -B_2s & 0 \end{vmatrix}=F(s)\times B_2s$$

$$\Delta=\begin{vmatrix} M_1s^2+(B_1+B_2)s & -B_2s \\ -B_2s & B_2s+K_2 \end{vmatrix}=\left[M_1s^2+(B_1+B_2)s\right]\times\left[B_2s+K_2\right]-(B_2s)^2$$

$$=M_1B_2s^3+K_2M_1s^2+B_1B_2s^2+B_1K_2s+B_2K_2s$$

Therefore, $$X_1(s)=\frac{\Delta_1}{\Delta}=\frac{F(s)\times(B_2s+K_2)}{\Delta}$$

and $$X_2(s)=\frac{\Delta_2}{\Delta}=\frac{F(s)\times B_2s}{\Delta}$$

Thus, the transfer function of the given mechanical system is given by

$$\frac{X_1(s)}{F(s)}=\frac{(B_2s+K_2)}{\Delta}$$

$$\frac{X_2(s)}{F(s)}=\frac{B_2s}{\Delta}$$

where $\Delta = M_1B_2s^3+K_2M_1s^2+B_1B_2s^2+B_1K_2s+B_2K_2s$

1.7.4 A Simple Rotational Mechanical System

According to D'Alembert's principle, "The algebraic sum of the externally applied torques to any body is equal to the algebraic sum of opposing torques restraining motion produced by the elements present in the body."

A simple rotational mechanical system with all the basic elements and its free body diagram is shown in Fig.1.25 and Fig.1.26 respectively.

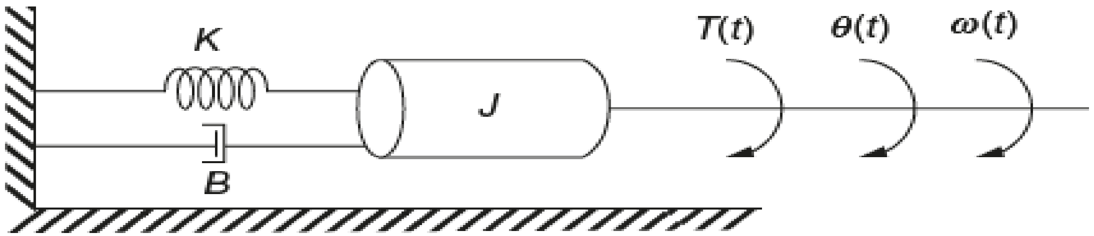


Fig. 1.25 | A simple rotational mechanical system

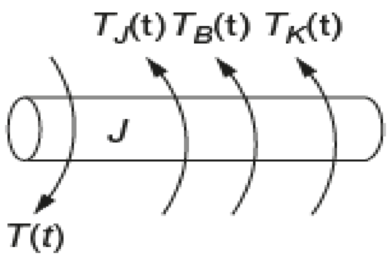


Fig. 1.26 | Free body diagram

Let $T(t)$ be the external torque applied to the given system,

$T_J(t) = J\dfrac{d\omega(t)}{dt}$ be the opposing torque produced by the element mass J,

$T_B(t) = B\omega(t)$ be the opposing torque produced by the element damper B and

$T_K(t) = K\int \omega(t)dt$ be the opposing torque produced by the element spring K.

Using D'Alembert's principle, $T(t) = T_J(t) + T_B(t) + T_K(t)$

$$T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) + \frac{1}{k}\int \omega(t)dt$$

The free body diagram of the system denoting the applied and opposing torques is shown in Fig. 1.26.
Restraining torques are given below:

Inertia torque $T_J(t) = J\frac{d\omega(t)}{dt}$

Damping torque $T_B(t) = B\omega(t)$

Spring torque $T_K(t) = K\int \omega(t)dt$

Introduction to Analogous System

Two equations of similar form are defined as analogous systems. The advantage of analogous system is that, if the response of one system is known, then the other system is also known without actually solving it.

Generally, mechanical systems are converted into analogous electrical system. There is a similarity between the equilibrium equations (integro-differential equations) of electrical systems and mechanical systems. Due to this, an electrical equivalent can be drawn to a mechanical system.

Example of Analogous Systems

A transformer is analogical to a gear. The transformer adjusts its "voltage-to-current" ratio (V/I) based on the load applied on the transformer that is similar to that of a gear mechanism, by which it adjusts the "torque-to-speed" ratio (T/ω) based on the load requirement.

Advantages of Electrical Analogous System

The following are the advantages of electrical analogous system:

(i) Standard symbols are available in electrical systems such as resistance, inductance, capacitance, etc.

(ii) Standard and simple laws are available in electrical system that makes it easier for calculation.

Example: Kirchhoff's laws, Thévenin's theorem, etc.

(iii) It is easy to analyze a system for different parameter values in an electrical system, because R, L and C cannot be varied and their response can be seen.

In an electrical system, the dual networks are analogous systems and both are in electrical form. The analogous parameters in dual network are listed in Table 1.5.

Table 1.5 | Analogous parameters in dual network

Loop analysis	Nodal analysis
Voltage (V)	Current (I)
Current (I)	Voltage (V)
Charge (q)	Flux (Φ)
Resistance (R)	Conductance (G)
Capacitance (C)	Inductance (L)
Inductance (L)	Capacitance (C)

1.8.2 Force–Voltage Analogy

Consider a simple translational mechanical system as shown in Fig. 1.28.

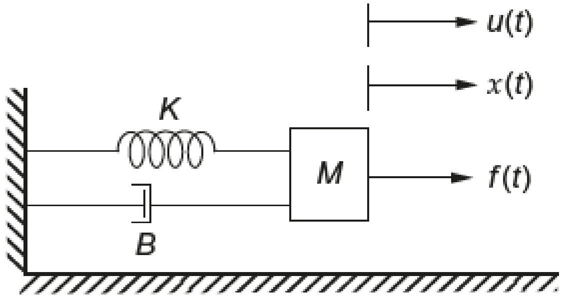


Fig. 1.28 | Translational mechanical system

Using D'Alembert's principle, we have
Sum of the applied forces = sum of the opposing forces

$$f(t) = M\frac{du(t)}{dt} + Bu(t) + K\int u(t)\,dt \tag{1.6}$$

Consider a series RLC circuit as shown in Fig. 1.29.

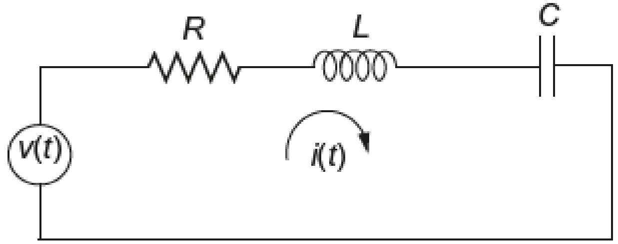


Fig. 1.29 | Series RLC circuit

Using KVL, the integro-differential equations can be written as

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \tag{1.7}$$

Rearranging Eqs. (1.6) and (1.7), we obtain

$$f(t) = M\frac{du(t)}{dt} + Bu(t) + K\int u(t)\,dt$$

$$v(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt$$

Since the force of mechanical system is made analogous to the voltage of electrical system, the above two equations are represented as analogous system. This analogy is known as force–voltage (or) *F–V* analogy. The analogous parameters in *F–V* system are given in Table 1.6.

Table 1.6 | *F–V* analogous parameters

Translational system	Electrical system
Force (f)	Voltage (v)
Velocity (u)	Current (i)
Displacement (x)	Charge (q)
Mass (M)	Inductance (L)
Damping coefficient (B)	Resistance (R)
Spring constant (K)	1/Capacitance (C)

Force–Current Analogy

Consider a simple parallel RLC Circuit as shown in Fig. 1.30.

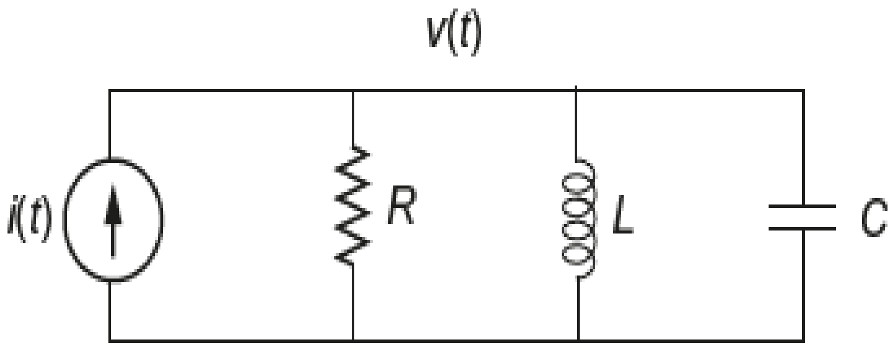


Fig. 1.30 | Parallel RLC circuit

Using KCL, the integro-differential equations can be written as follows:

$$i(t)=\frac{v(t)}{R}+C\frac{dv(t)}{dt}+\frac{1}{L}\int v(t)dt$$

$$i(t)=Gv(t)+C\frac{dv(t)}{dt}+\frac{1}{L}\int v(t)dt \qquad (1.8)$$

The force equation for translational mechanical system is

$$f(t)=M\frac{du(t)}{dt}+Bu(t)+K\int u(t)dt \qquad (1.9)$$

Rearranging Eqs. (1.8) and (1.9), we obtain

$$i(t)=C\frac{dv(t)}{dt}+Gv(t)+\frac{1}{L}\int v(t)dt$$

$$f(t)=M\frac{du(t)}{dt}+Bu(t)+K\int u(t)\,dt$$

Since the force of mechanical system is made analogous to the current of electrical system, the above two equations are represented as analogous system. This analogy is known as force–current (or) *F–I* analogy. The analogous parameters in *F–I* system are given in Table 1.7.

Table 1.7 | *F–I* analogous parameters

Translational System	Electrical System
Force (f)	Current (i)
Velocity (u)	Voltage (v)
Displacement (x)	Flux (Φ)
Mass (M)	Capacitance (C)
Damping coefficient (B)	Conductance (G)
Spring constant (K)	1/Inductance (L)

Table 1.8 | Analogous of *F–V* and *F–I* to translational mechanical system

Translational Mechanical System	*F-V* Analogy	*F-I* Analogy
Force (f)	Voltage (v)	Current (i)
Velocity (u)	Current (i)	Voltage (v)
Displacement (x)	Charge (q)	Flux (Φ)
Mass (M)	Inductance (L)	Capacitance (C)
Damping coefficient (B)	Resistance (R)	Conductance (G)
Spring constant (K)	1/Capacitance (C)	1/Inductance (L)

Torque–Voltage Analogy

Consider a simple rotational mechanical system as shown in Fig.1.32.

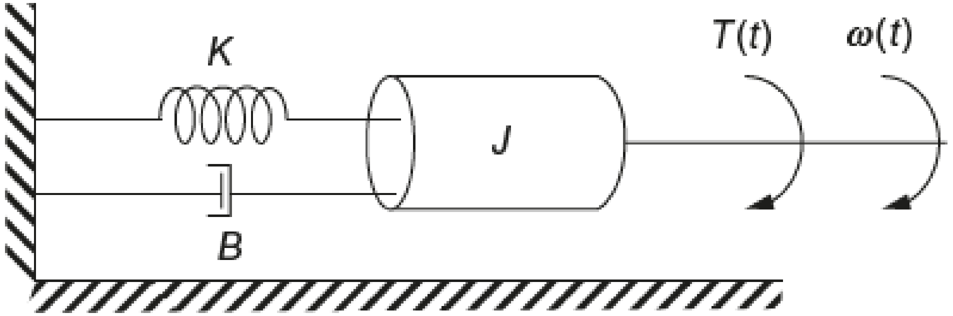


Fig. 1.32 | Rotational mechanical system

Using D'Alembert's principle, we have
Sum of the applied torques = sum of the opposing torques

$$T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) + K\int \omega(t)\,dt \tag{1.10}$$

Consider a series RLC circuit as shown in Fig. 1.33.

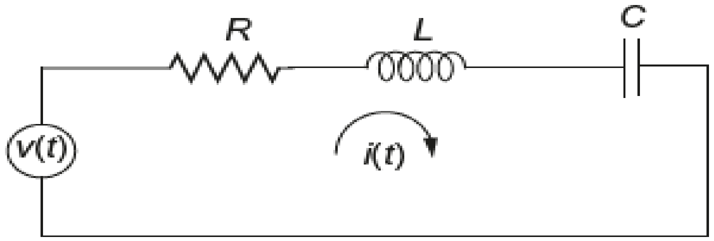


Fig. 1.33 | Series RLC circuit

Applying KVL, we obtain

$$v(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)\,dt \tag{1.11}$$

Since the torque of the mechanical system is made analogous to the voltage of the electrical system, Eqs. (1.10) and (1.11) are represented as analogous system. This analogy is known as *torque–voltage* (or) *T–V analogy*. The analogous parameters in *T–V* system are given in Table 1.9.

Table 1.9 | *T–V* analogous parameters

Rotational System	Electrical System
Torque (T)	Voltage (v)
Angular velocity (ω)	Current (i)
Angular displacement (θ)	Charge (q)
Moment of inertia (J)	Inductance (L)
Rotational damping (B)	Resistance (R)
Rotational spring constant (K)	1/Capacitance (C)

Torque–Current Analogy

Consider a simple rotational mechanical system as shown in Fig. 1.34.

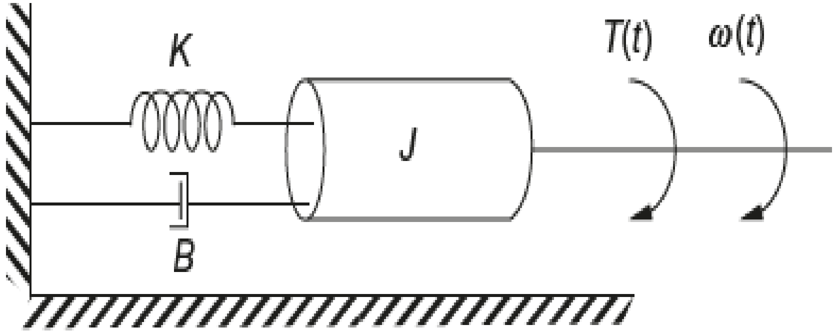


Fig. 1.34 | Rotational mechanical system

Using D'Alembert's principle, we have

$$T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) + K\int \omega(t)dt \tag{1.12}$$

Consider parallel RLC Circuit as shown in Fig. 1.35.

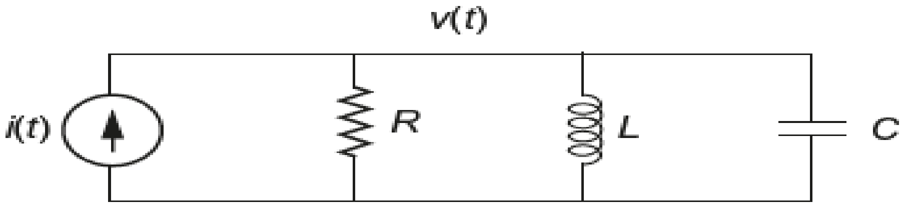


Fig. 1.35 | Parallel RLC circuit

Applying KCL, we obtain

$$i(t) = C\frac{dv(t)}{dt} + \frac{v(t)}{R} + \frac{1}{L}\int v(t)dt$$

$$= C\frac{dv(t)}{dt} + Gv(t) + \frac{1}{L}\int v(t)\,dt \tag{1.13}$$

Since the torque of the mechanical system is made analogous to the current of the electrical system, Eqs. (1.12) and (1.13) are represented as analogous system. This analogy is known as torque–current (or) *T–I* analogy. The analogous parameters in *T–I* system are given in Table 1.10.

Table 1.10 | *T–I* analogous parameters

Rotational Mechanical System	*T-I* Analogous
Torque (T)	Current (i)
Angular velocity (ω)	Voltage (v)
Angular displacement (θ)	Flux (Φ)
Moment of inertia (J)	Capacitance (C)
Rotational spring constant (K)	1/Inductance (L)
Rotational damping (B)	Conductance (G)

The analogous variables for rotational mechanical system and electrical system are listed in Table 1.11.

Table 1.11 | Analogous of *T–V* and *T–I* to rotational mechanical system

Rotational Mechanical System	*T–V* Analogy	*T-I* Analogy
Torque (T)	Voltage (v)	Current (i)
Angular velocity (ω)	Current (i)	Voltage (v)

(Continued)

Table 1.11 | (Continued)

Rotational Mechanical System	*T-V* Analogy	*T-I* Analogy
Angular displacement (θ)	Charge (q)	Flux (Φ)
Moment of inertia (J)	Inductance (L)	Capacitance (C)
Rotational damping (B)	Resistance (R)	Conductance (G)
Rotational spring constant (K)	1/Capacitance (C)	1/Inductance (L)

References

1. **Control System Engineering by** I J Nagarath & M Gopal
2. **Control System Engineering by** Shalivahanan
3. **Modern Control Engineering by** Ogata


THANK YOU!